Edit Distance: Sketching, Streaming and Document Exchange

Djamal Belazzougui
CERIST, Algeria

Qin Zhang
IU Bloomington

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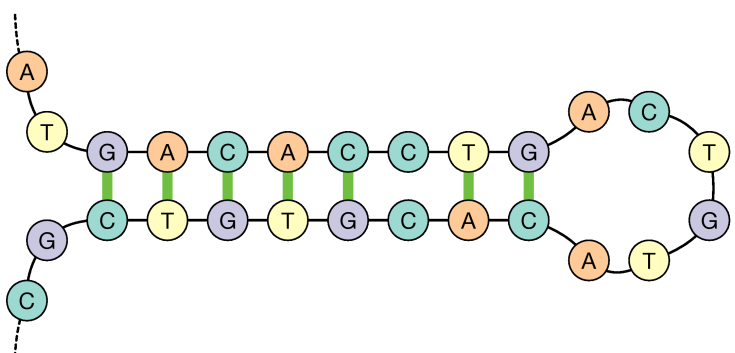

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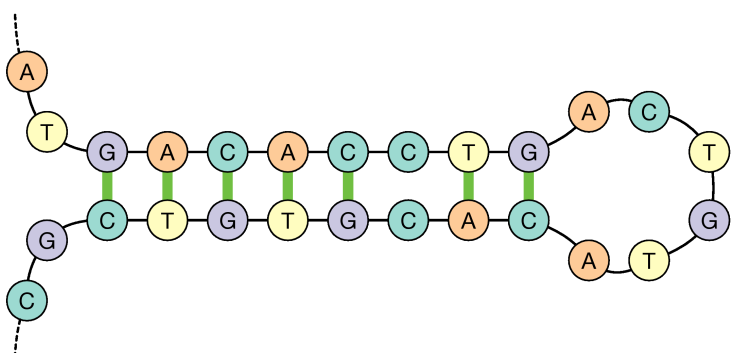

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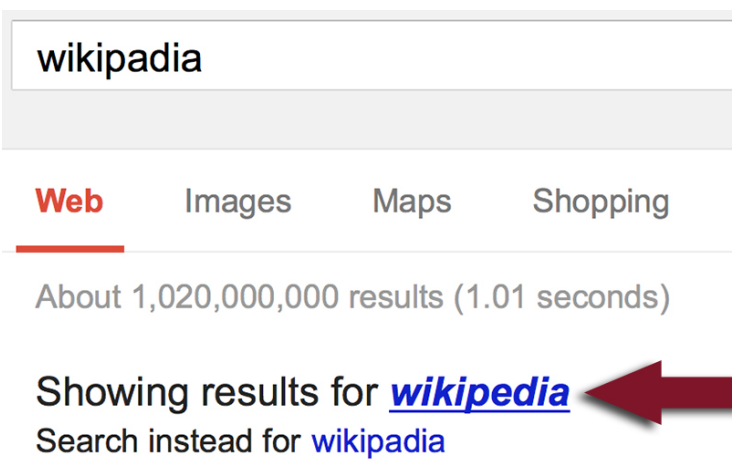

automatic spelling correction

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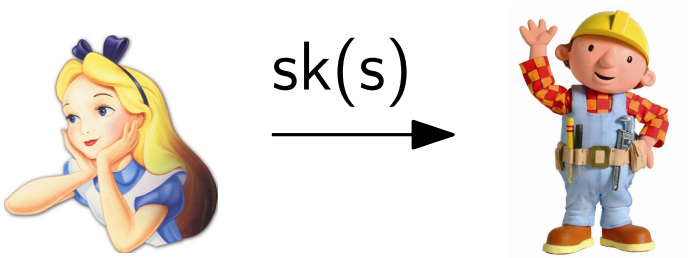

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App: remote file sync;
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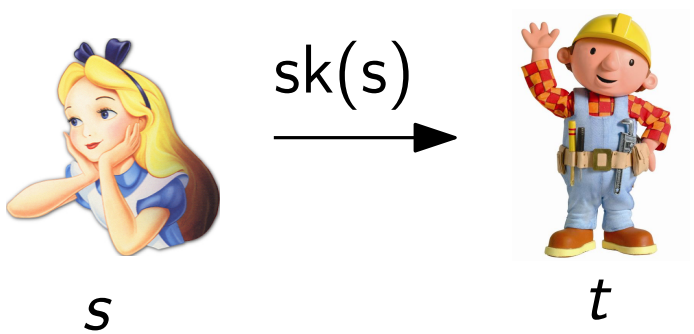

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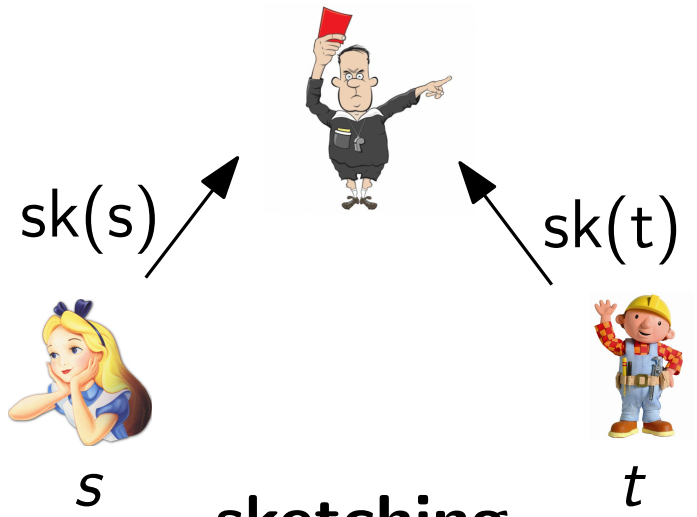

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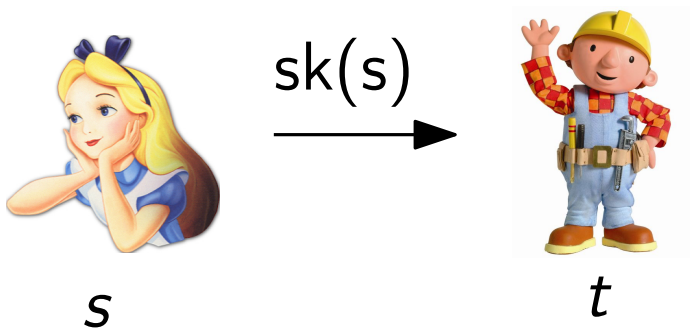

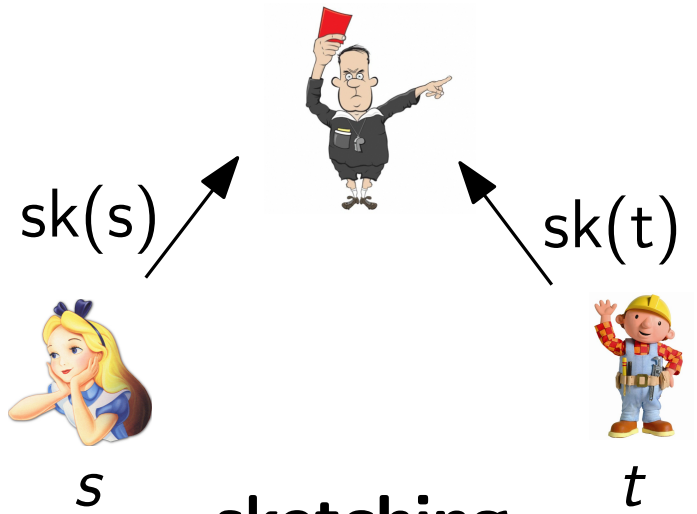

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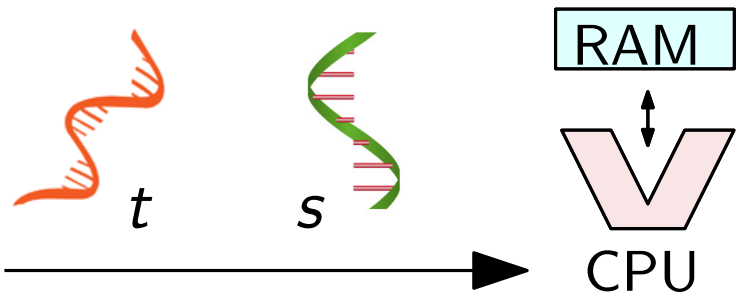

Previous and our results

problem	comm. / size / space (bits)	running time	rand. or det.	ref.
document-exchange	$O(K \log n)$	$n^{O(K)}$	D	[23]
	$O(K \log(n/K) \log n)$	$\tilde{O}(n)$	R	[18]
	$O(K \log^2 n \log^* n)$	$\tilde{O}(n)$	R	[19]
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	$O(K^2 \log n)$	$\tilde{O}(n)$	R	[8]
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sketching	$O(K^8 \log^5 n)$	$\tilde{O}(K^2 n)$ (enc.), poly$(K \log n)$ (dec.)	R	**new**
streaming	$O(K^8 \log^5 n)$	$\tilde{O}(K^2 n)$	R	**new**
simultaneous-streaming	$O(K^6 \log n)$	$\tilde{O}(n)$	R	[8]
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K: distance threshold; n: input size. For simplicity, assuming $K < n^{0.1}$

- Information theoretic optimal communication for $K \leq 2^{\sqrt{\log n}}$ under almost linear encoding/decoding time for doc-exchange.
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IMS scheme

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Main Tool:
CGK Embedding

Our main tool – CGK embedding

- The CGK embedding

(Chakraborty, Goldenberg, Koucky, STOC'16
Similar idea by Saha, FOCS'14)

$f : s \in \{0,1\}^n \to s' \in \{0,1\}^{3n}$.

Two counters i and j both initialized to 1. For $j = 1, 2, \ldots$ steps:

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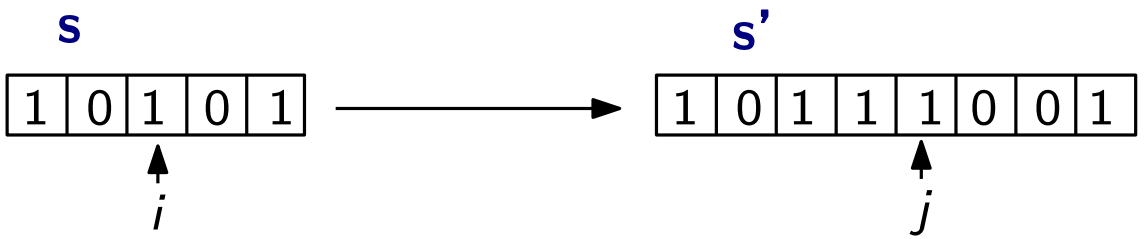

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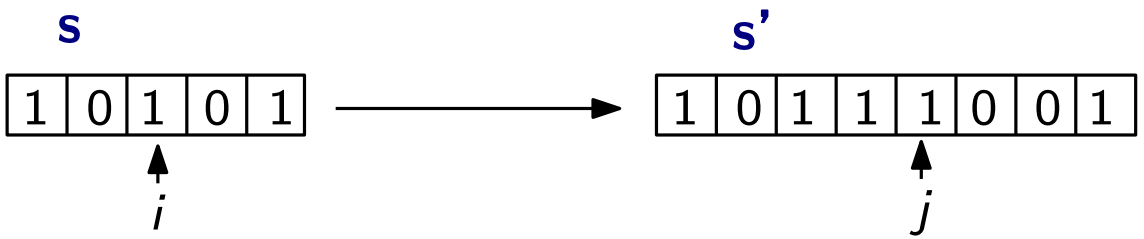
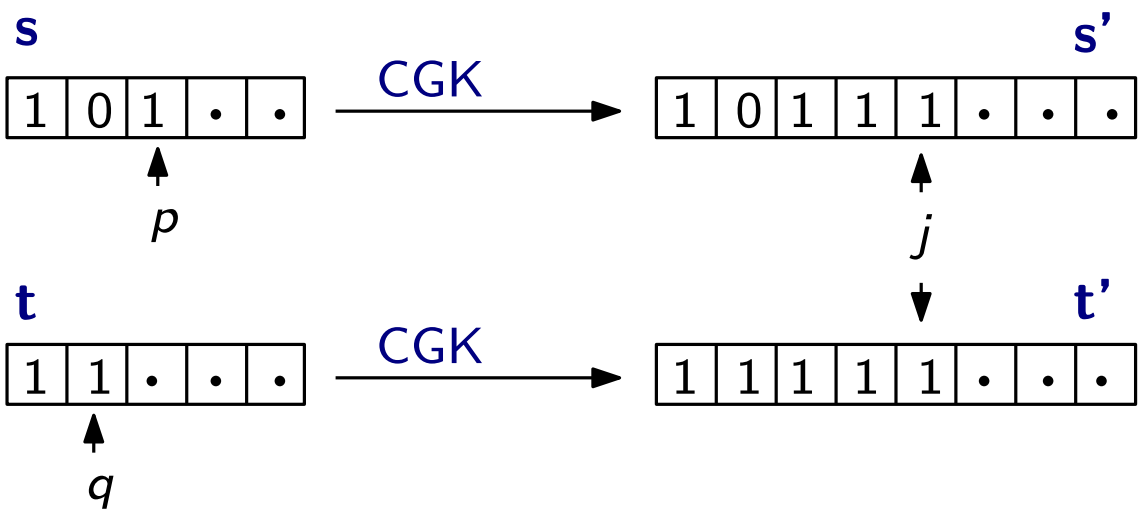


- Property

If $ed(s,t) = k$, then $k/2 \leq ham(f(s), f(t)) \leq O(k^2)$ w.pr. 0.99

CGK as a random walk

- CGK $\rightarrow$ a random walk on two strings



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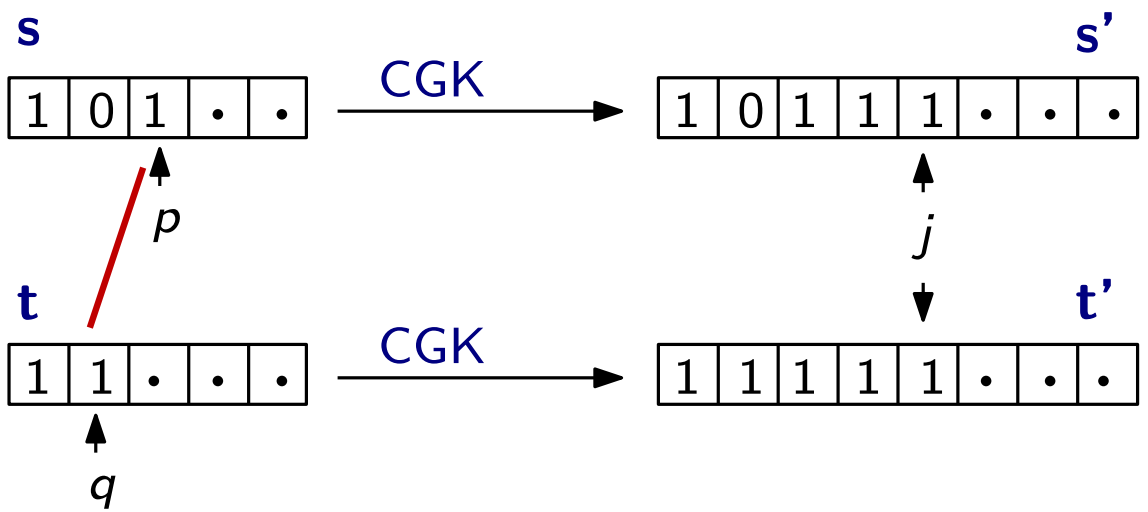
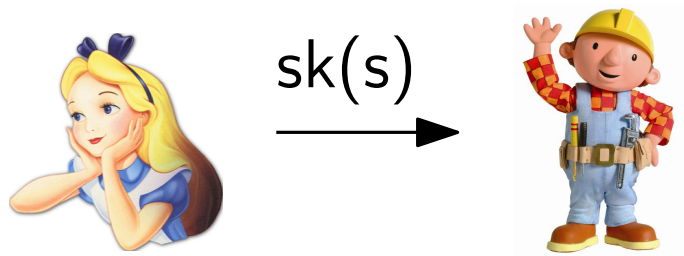


- The shift $(p - q)$ is a random walk on the line.

Document Exchange



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Warning: I will cheat in multiple places

Technique overview: document exchange

- **Main idea**: If we can find $\leq K$ pairs of blocks in s and t each of size K^{99}, such that they contain all the edits, then IMS gives $O(K(\log^2 K))$. (recall IMS gives $O(K \log n \log(n/K))$)

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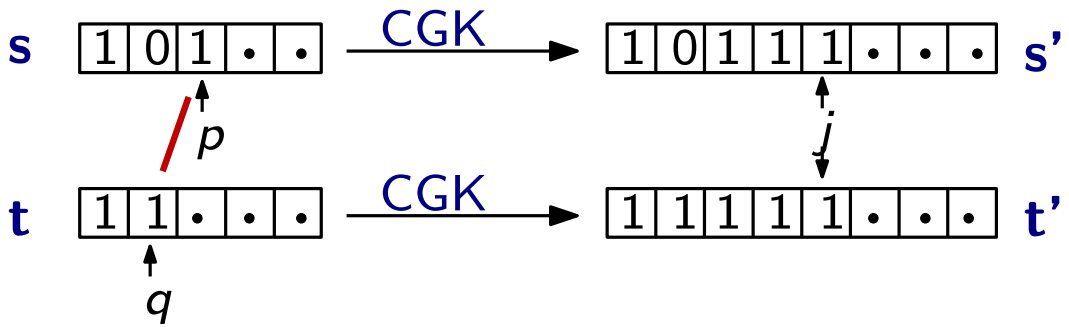
  - Can reduce $O(K^2)$ pairs to $O(K)$, by removing long common periodic substrings.
  - Not easy: everything has to be done using one-way comm.!

Technique overview: document exchange (cont.)



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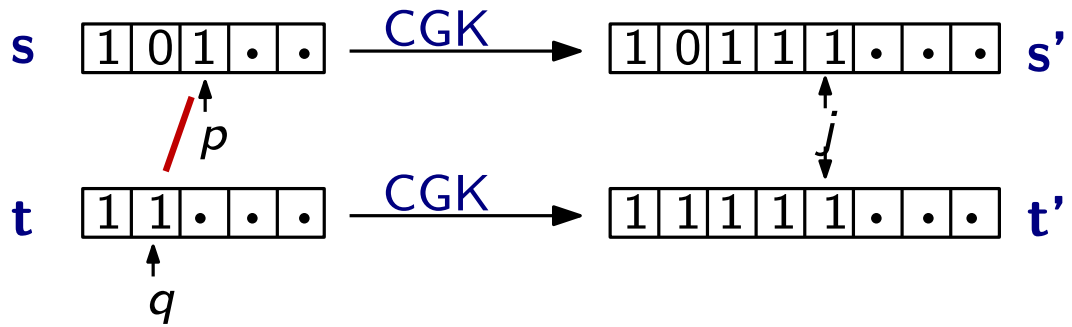

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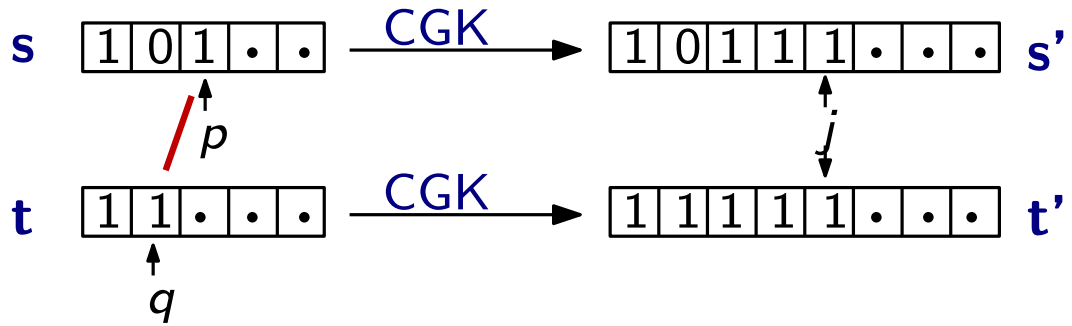

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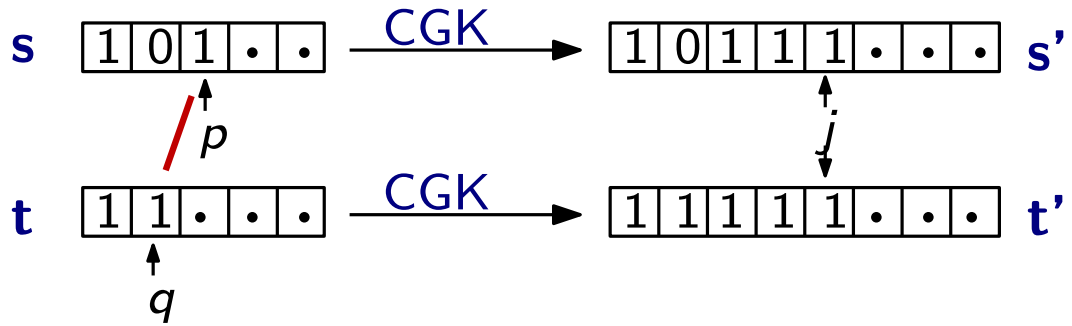

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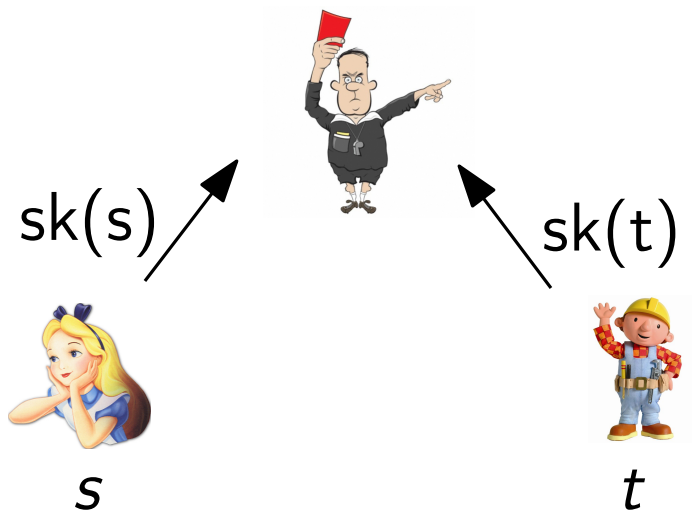
Recall our main idea: If we can find $\leq K$ pairs of blocks in s and t each of size K^{99}, such that they contain all the edits, then IMS gives $O(K(\log^2 K))$. (Other steps cost $O(K \log n)$)

Sketching



App: distributed similarity join

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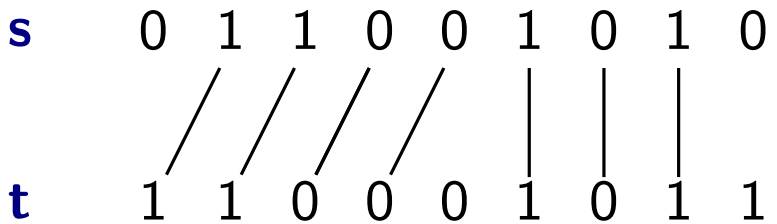

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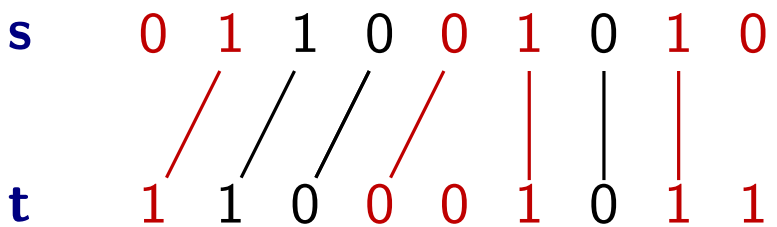

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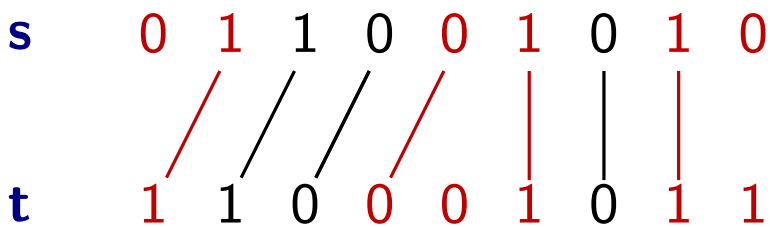

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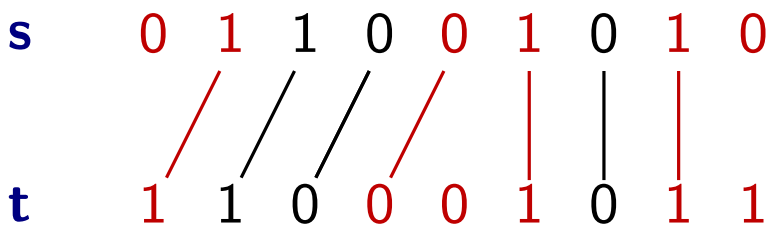

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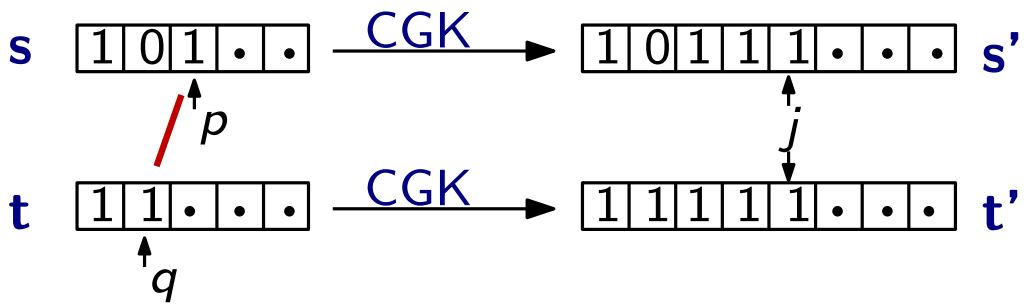
- Given alignments $\mathcal{A}_1, \ldots, \mathcal{A}_\rho$, letting $\mathcal{I} = \bigcap_{j \in [\rho]} \mathcal{A}_j$

  Main idea: if $\exists$ an optimal alignment that goes through all edges in $\mathcal{I}$, then we can obtain an optimal alignment using $sk(\mathcal{A}_1), \ldots, sk(\mathcal{A}_\rho)$

Technique overview: sketching (cont.)

- CGK embedding naturally gives an alignment.



The random walk state sequence $((p_1, q_1), (p_2, q_2), \ldots)$ contains an alignment $\mathcal{A}$, which can be constructed in a greedy way, and $sk(\mathcal{A})$ has size $\text{poly}(K, \log n)$.

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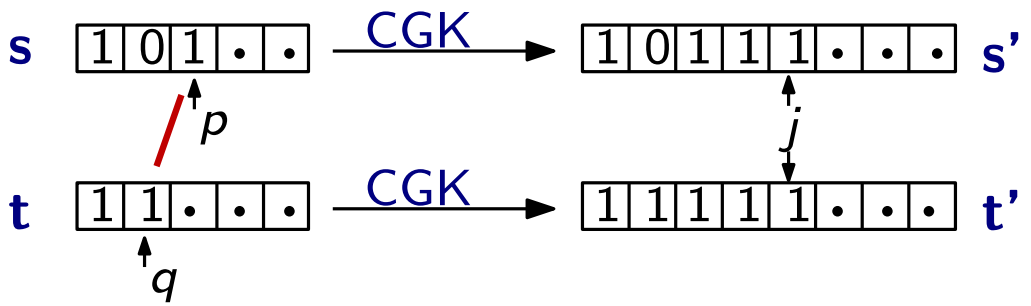

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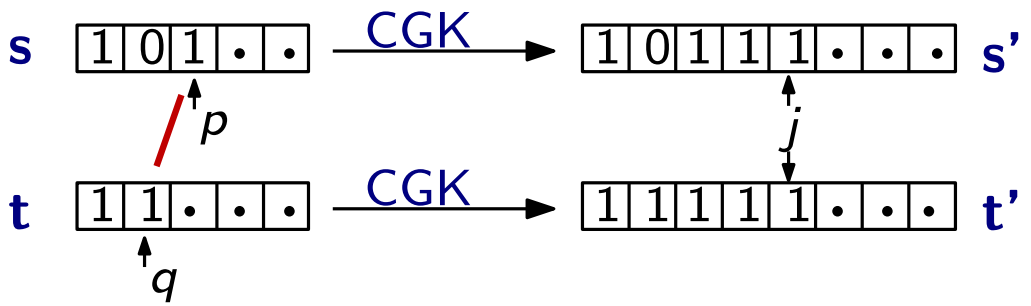

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- $sk(\mathcal{A}_i) \Leftrightarrow$ differences between s' and t' in the ham-space for which efficient sketching algo is known.

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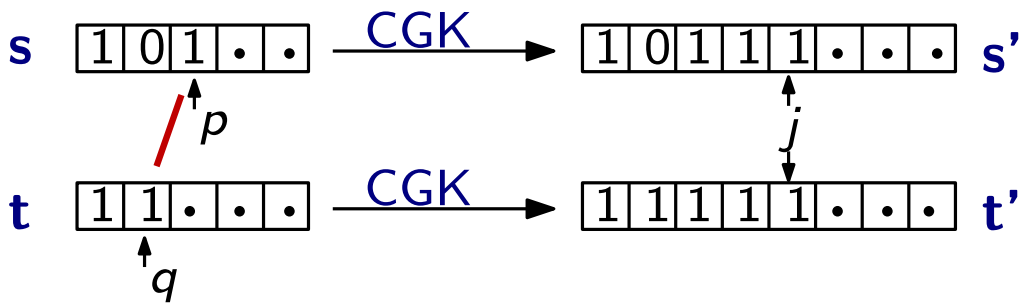
- CGK embedding naturally gives an alignment.



The random walk state sequence $((p_1, q_1), (p_2, q_2), \ldots)$ contains an alignment $\mathcal{A}$, which can be constructed in a greedy way, and $sk(\mathcal{A})$ has size poly$(K, \log n)$.

- **Key lemma**: Can show if we take $\rho =$ poly$(K, \log n)$ random walks which give alignments $\mathcal{A}_1, \ldots, \mathcal{A}_\rho$, then there is an optimal alignment contains $\mathcal{I} = \bigcap_{j \in [\rho]} \mathcal{A}_j$
- $sk(\mathcal{A}_i) \Leftrightarrow$ differences between s' and t' in the ham-space for which efficient sketching algo is known.
- Additional structures needed for the reverse mapping (ham-space $\rightarrow$ edit-space) to find all the edits.

Conclusion and open problems

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  - Information theoretic optimal communication (for small K) under almost linear encoding/decoding time for document exchange.
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  - Is it possible to derandomize our algorithm for doc-exchange to obtain a better error-correcting code for edit distance?

Thank you!
Questions?

Technique overview: sketching (cont.)

Key lemma: Can show if we take $\rho = \text{poly}(K, \log n)$ random walks which give alignments $\mathcal{A}_1, \ldots, \mathcal{A}_\rho$, then there is an optimal alignment contains $\mathcal{I} = \bigcap_{j \in [\rho]} \mathcal{A}_j$

- **Anchor**. Given ρ random walks generated according to the CGK embedding, we say a pair (u, v) is an anchor if $s[u] = t[v]$, and all the ρ random walks pass (u, v) .
- **Claim**: W.pr. $1 - 1/n^2$, there is an optimal alignment going through all anchors.
- **Proof idea**: We focus on a "greedy" optimal matching $\mathcal{O}$. Suppose on the contrary that $\mathcal{O}$ does not pass an anchor (u, v), then we can find a matching M in the left neighborhood of (u, v) which may "mislead" a random walk, that is, with a non-trivial probability the random walk will "follow" M and consequently miss (u, v).