

CSCI 503B: Homework 5

Solve ALL questions.

1. Solve or bound the following two recurrences:
 - (a) $T(1) = 1$ and $T(n) = 2T(n/4) + n \log n$ for $n > 1$.
 - (b) $T(1) = T(2) = T(3) = 1$ and $T(n) = 2T(n - 3)$ for $n > 3$. (Hint: use a recursion tree. What is the depth of the tree? How many nodes per level?)
2. I go on a road trip and cover n miles in stages. Each stage can be 2, 3, or 4 miles. My stages thus form a sequence such as 2, 3, 2, 4, 3, 3, ... which adds up to n . Give a dynamic program to find out how many different ways are there of doing this trip. Does this look like any other problem that we have seen? (Answer: yes, very much!) Make sure that you handle the differences between the two problems.
3. Recall the knapsack problem. We have gold dust, silver dust, and iron dust (of limited quantities) in a room, and a limited size knapsack. Clearly gold is more valuable per gram than silver, which is more valuable than iron. I fill my knapsack as follows: I grab as much gold as I can (until there is no more or my knapsack is full), then as much silver that I can take (if space left) and then iron. Show that this greedy solution is optimal. I would use proof by contradiction, but you can do induction, or perhaps even other things...
4. Consider Huffman coding. The frequencies of the elements are 1, 2, 3, 5, 8, 13, etc., following the Fibonacci series. What does the tree look like? Prove. (Recall: I did this in class.)
5. I have a list of n numbers whose longest increasing subsequence is known to be of length $n - c$ for a constant c (longest increasing subsequence is an increasing subsequence of maximum length in a list. For instance, for 1 2 3 4 5 9 7 8 6 it is 1 2 3 4 5 7 8.) What algorithm, among those we covered in class (insertion sort, mergesort, quicksort) would you use in order to sort these numbers using the smallest number of operations? Argue. How many operations did you use to sort the list?
6. Prove: (i) 3^n is not $O(2^n)$. (ii) $(n/2)^{n/2} = O(n!)$ (Hint: write out the products!)