CSCI 241H:

HOMEWORK 3

Show your work.

1. You are given the following about the sizes of sets A and B: $|A| = k$; $|B| = l$, and $|A - B| = m$. What is the size of $P(A \cap B)$? Note that $P(S)$ refers to the power set of set S.

   To do that you have to argue about the size of $A \cap B$. Since this set is all x such that $x \in A \wedge x \in B$, its size is $l - m$. You should argue this a bit more formally by arguing that $A - B$ and $A \cap B$ are disjoint, and together they make up A. The size of the power set then is 2^{k-m}

2. Is it true that, if $B \subseteq A$, then $B \cap C \subseteq A \cap C$ for all sets C? Prove.

   We have that any $x \in B$ is also in A. Thus anything in $B \cap C$ is in B and in C, and thus in A and in C. The claim follows.

3. Show the following.

   (a) $(A - B) - C \subseteq A - C$

   (b) $(A - C) \cap (C - B) = \emptyset$

   (c) $(A \cup B) \subseteq (A \cup B \cup C)$

   (a) Anything in $(A - B) - C$ is in A, but not in B, and not in C. So, these are all x such that $x \in A \wedge x \notin B \wedge x \notin C$. Using rules of inference, these imply $x \in A \wedge x \notin C$. Ths proves the claim.

   (b)Anything that satisfies this satisfies:

   $x \in A \wedge x \notin C \wedge x \in C \wedge x \notin B$.

   This implies $x \in C \wedge x \notin C$, which is false.

   (c) Any x that satisfies the LHS satisfies $x \in A \vee x \in B$. But that implies, by rules of inference, $x \in A \vee x \in B \vee x \in C$. Thus the claim follows.

4. Show that there are as many numbers that can be written as decimal fractions (i.e., of the form $a.b$ where a is an integer and b is a positive integer), as there are nonnegative integers and vice versa.

   I will first show equivalence to integers. Since we showed equivalence of the cardinality of integers to positive integers, the claim follows. Any integer a is a decimal fraction $a.0$. So this direction is easy. Now consider any decimal $a.b$, where a and b are integers. This is very similar to rational numbers: write a and b in base 9, write the decimal point as the digit 9. Then, any decimal fraction will map to a distinct integer thus their number cannot be more than integers.