CSCI 241H:

HOMEWORK 2

Show your work.

1. Write the following in predicate logic. Don't forget the quantifiers; explain what each predicate means.

   (a) Every student who takes 241 either comes to class regularly, or borrows the notes from someone who comes to class regularly.
   Let $R(x)$ denote "x comes to class regularly." $B(x, y)$ is "x borrows notes from from y." $T(x)$ is "x takes 241." Then,

   $$\forall x(T(x) \rightarrow (R(x) \vee \exists y(R(y) \wedge B(x, y)))$$

   (b) There is no one in the world who is loved by everybody.
   $L(x, y)$ means x loves y.

   $$\neg\exists y \forall x L(x, y)$$

   (c) There is such a man in this group that no one is taller than he is.
   $T(x, y)$: x is taller than y. G is our group.

   $$\exists x \in G \ \neg\exists y \in G \ T(y, x)$$

2. Show that

   $$\forall x P(x) \vee \forall x Q(x) \neq \forall x(P(x) \vee Q(x))$$

   This is best done with an example. Let $P(x)$ stand for x is male. $Q(x)$ is x is female. Any person is either male or female, thus RHS is true. LHS is not, since it is not true that either everyone is male or everyone is female. This is a place where a counterexample works best.

3. Are the following two expressions equivalent? Argue.

   $$\forall x(P(x) \rightarrow Q(x))$$

   and

   $$(\forall x P(x)) \rightarrow (\forall x Q(x))$$

There are multiple ways of doing this. I will show that each statement implies the other – call the first one A and the second B. I will show that (1) $A \to B$ by showing $\neg(A \to B)$ is false, then (2) do the same for $B \to A$.

So let's start with (1) $\neg(A \to B)$, which is $A \wedge \neg B$. This is

$\forall x(P(x) \to Q(x)) \wedge \neg(\forall x P(x) \to \forall x Q(x))$, which is

$\forall x(P(x) \to Q(x)) \wedge \forall x P(x) \wedge \exists x \neg Q(x)$

We can instantiate the existential quantifier with one particular value, call a. But then we can instantiate all univeral quantifiers with a as well, since we can instantiate them with anything. This gives us

$(P(a) \to Q(a)) \wedge P(a) \wedge \neg Q(a)$

Rewriting the last two terms, this is

$(P(a) \to Q(a)) \wedge \neg(P(a) \to Q(a)) = F$

Thus $A \to B$ must be true. Now let's prove $B \to A$ by showing $\neg(B \to A) = B \wedge \neg A$ is false, that is, (2).

$B \wedge \neg A = (\forall x P(x) \to \forall x Q(x)) \wedge \neg(\forall x(P(x) \to Q(x))) = (\forall x P(x) \to \forall x Q(x)) \wedge (\exists x \neg(P(x) \to Q(x)))$

We can again instantiate the existential statement with b, which can be used to instantiate the universal statement, thus giving

$(P(a) \to Q(a)) \wedge \neg(P(a) \to Q(a)) = F$. Thus we are done.

4. Page 80, question 35 from your book:

   If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he woul be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.

   Notation:

   W(x): x is willing to prevent evil

   A(x): x is able to prevent evil

   P(x): x prevents evil

   I(x): x is impotent

   M(x): x is malevolent

   E(x): x exists.

S stands for Superman – you can actually do without it.

Let's write the givens:

1: $(A(S) \wedge W(S)) \rightarrow P(S)$

2. $\neg A(S) \rightarrow I(S)$

3. $\neg W(S) \rightarrow M(S)$

4. $\neg P(S)$

5. $E(S) \rightarrow (\neg M(S) \wedge \neg I(S))$

Let's keep going.

6. $\neg A(S) \vee \neg W(S)$ from 1, 4

7. $M(S) \vee I(S)$ from 2, 3, 6, use resolution twice.

8. $\neg(\neg M(S) \wedge \neg I(S))$ from 7

9. $\neg E(S)$ 8, 5