CSCI 241H:

HOMEWORK 10

Solve the first four questions. Show your work.

1. Use the pigeonhole principle to show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs which add up to 11. Hint: List all pairs which add up to 11.
   Group the integers into five groups of two such that each group adds up to 11. eg, 1 and 10, 2 and 9, etc. If you pick 7 elements, by the pigeonhole principle, two groups will have two elements in them, so there will be two pairs which add up to 11.

2. If I'm teaching a class of 301 people, there must be at least 3 people who got 80 in the exam. True or false? Argue.
   False. What if everyone got 100?

3. In a group of 7 girls and 9 boys, how many ways are there to pick a team of 4 in which both genders are represented? Add up all ways: one girl, three boys: 7*84. Two each: 21 *36. One boy, three girls: 56 * 9.

4. If I were to change English so that vowels and consonants would alternate (for instance, "pirate" would be a valid word while "private" would not), how many seven-letter words could be possible?
   If I start with a vowel, 5 * 5 * 5 * 5 + 21 * 21 * 21. If I start with a consonant, 21 * 21*21*21*5*5*5.

5. I have 5 white and 10 red balls. I want to arrange them in a line so that no two white balls are next to one another. How many ways are there of doing this? Note that I cannot tell one red or white ball from another.

   I put in WRWRWRWRW first. Then I can throw my remaining 5 red balls into 6 locations with respect to the whites. As we did in class, this is $C(11, 6)$.

6. Prove using the pigeonhole principle that, at a party with more than 2 people, at least two people know the same number of people who are at the party.

7. Prove using the pigeonhole principle that an undirected graph with more than one vertex has at least two vertices with the same degree.