

## CSCI 241H, Midterm

Recall that a  $d$ -regular undirected graph is one where each vertex has degree  $d$ .

- (1 pt) (answer before you start) Hello! How are you feeling?
- (18 pts) Let  $G = (V, E)$  be a 2-regular bipartite graph. Show that it has a matching of size  $|V|/2$ . Alternatively, let there be  $n$  people on an island where each woman is compatible with (ie, she can marry) exactly two men, and each man is compatible with exactly two women. If we allow only compatible people to marry, and a person can only marry exactly one person of the opposite sex, show that we can end up with  $n/2$  marriages.
- (18 pts) You are given an undirected connected graph  $G$ . Show that the two following two statements about an edge  $(a, b)$  in  $G$  are equivalent, i.e., they imply each other (in other words, show an if and only if relationship between them). (a) There is no simple cycle (ie, one that does not repeat an edge) in  $G$  that contains both  $a$  and  $b$ . (b) If we remove  $(a, b)$ ,  $G$  becomes disconnected.
- (27 pts) Prove the following statements by induction:
  - Any set of size  $n$ , for  $n \geq 3$ , has  $n(n-1)(n-2)/6$  subsets of size 3.
  - For  $n > 6$ ,  $3^n < n!$
  - $n^2 - 1$  is divisible by 8 for odd positive  $n$ .
- (15 pts) Let  $f(x) = O(g(x))$ . Is it true that  $2^{f(x)} = O(2^{g(x)})$ ? Prove.
- (21 pts) One line answers.
  - What does NP stand for? Do not explain.
  - Can a 5-regular graph have 15 vertices? Argue in one line.
  - What is the asymptotic complexity of bubble sort? Just write the function.
- (2 pts)(answer after you're finished) How are you feeling now? Better, or worse?