

## CSCI 241H, Midterm 1

Pick any 5 questions; each question is 20 points. If you attempt all 6, I will drop the one where you got the lowest score.

1. Mary goes to the doctor if she's feeling neither energetic nor sharp. If she has a headache and blurred vision she has a migraine attack. If she does not have a headache she feels energetic. If she does not have blurred vision she feels sharp. She is not having a migraine attack. So, is it possible that she has gone to the doctor today? Prove your answer using rules of inference. Show every step, but you don't need to write down the name of the rule you're using (but feel free to put it down if you feel like it).
2. Consider the infinite set  $S = \{1, 2, 4, 8, 16, 32, 64, \dots\}$ . Argue that the cardinality of  $S$  is the same as the cardinality of the set of positive integers.
3. Argue that  $\exists xP(x) \wedge \exists xQ(x)$  is not equivalent to  $\exists x(P(x) \wedge Q(x))$ .
4. Use proof by contradiction to show that  $\sqrt{3}$  is irrational.
5. Argue that for any three sets  $A, B, C$ , if  $A \cap B \subseteq C$  then  $A - C \subseteq A - B$ . Hint: if  $x \in A - C$  then what can you say about  $x$ ? What do you need to show about  $x$ ? Do not give a proof by Venn diagram, even though you can use a Venn diagram for your own intuition.
6. Consider the recurrence  $f(i) = f(i - 1) + 2f(i - 2)$ . If  $f(0) = 1$  and  $f(1) = 2$ , give a closed form expression for  $f(n)$ . Note that a summation is not a closed form expression. Show your work.