CSCI 241H:
HOMEWORK

Solve the first four questions. Show your work.

1. $\overline{G}$, the complement of $G = (V, E)$ is a graph on vertex set V. It contains edge (i, j) if and only if G does not contain it in its edge set. For instance, the complement of a complete graph is just a bunch of nodes with no edges. Find a graph G on more than 3 nodes where G is isomorphic to its complement. Prove. Now, for a hard one: prove that the complement of a bipartite graph cannot be bipartite.

2. Show that a graph G with n vertices is connected if it has more than $(n-1)(n-2)/2$ edges.

3. How many vertices does a d-regular graph with m edges have? (recall that a $d-$ regular graph is one where each vertex has degree d.) Assume that the graph is undirected.

4. Show that in every undirected graph there is a path from every vertex of odd degree to some other vertex of odd degree. Hint: prove by contradiction.

5. 10.3, Questions 28, 52.