CSCI 241H:
HOMEWORK 7

Only questions 1-4 will be graded. Show your work.

1. Let T be a complete binary tree, with root node k. Let l_t be the number of nodes in the left subtree of k (the left subtree is the tree which has its left child as its root). Prove by induction that the number of nodes in l_t is 1 less than the number of leaves in the tree. Do not prove or argue what each quantity is, work on the relationship between them in an inductive fashion (you can do that for the base case). You will lose points for statements such as "well, the tree has q leaves because ..." or "we know that the size of the subtree is ...". I am not interested in how many leaves, or how many in the subtree – I want to be shown that one is 1 more than the other.

2. A $d-$regular graph is an undirected graph where each vertex is of degree exactly d. Is it possible to have a 3-regular graph on 5 nodes? Prove. How about on 6 nodes?

3. Prove that the sum of the indegrees of the vertices of a directed graph is equal to the sum of the outdegrees.

4. Show that in an undirected graph with at least two vertices there must be two vertices which have the same degree.

5. Let G be a bipartite graph where every vertex has the same degree (G is $d-$regular). Show that the sizes of the two partitions are equal.

6. Let G be a bipartite graph with n vertices and m edges. Show that $e \leq v^2/4$.

7. This is a tough one, I think: A Hamiltonian cycle is cycle through a graph (a sequence of edges which will leave you at your starting point), which visits every node exactly once). Show by induction that, for $d > 1$, any hypercube of degree (dimension) d has a Hamiltonian cycle.