CSCI 241H:
HOMEWORK 1

1. This problem is similar to Sudoku that we discussed in class. Consider a binary version of magic squares: you are given a 3×3 grid. Each of the 9 squares in your grid will contain a 0 or a 1: some are already given, some need to be filled out. The restriction is that the sum of the numbers in each row and each column will be exactly the same. For instance, a grid filled completely with 1s will satisfy this as all rows and columns will add up to 3.

   You are given that the square (1,1) is 1, (1,2) is 0 and (1,3) is 1. Write a formula such that this formula has a satisfying assignment if and only if this instance of magic squares has a solution (note that each rown and column must add up to 2).

   I do realize that this is an overkill for such a simple problem; the problem has a very easy, short solution. However, as one goes to bigger grids and non-binary cases, this will become much harder to do without a methodical approach, and I would like you to do the methodical approach with this super simple version. There are multiple ways of doing this, here's what I would do.

   Let $p(i, j)$ be T if $(i, j) = 1$, false otherwise. Then, we have values

$$p(1,1) \wedge p(1,3) \wedge \neg p(1,2)$$

   Now we know that each row should have exactly two 1s, so for $i = 2, 3$ we write

$$(p(i,1) \wedge p(i,2) \wedge \neg p(i,3)) \vee (p(i,1) \wedge \neg p(i,2) \wedge p(i,3)) \vee (\neg p(i,1) \wedge p(i,2) \wedge p(i,3))$$

   Then we write the same for the columns, for $j = 1 \ldots 3$

$$(p(1,j) \wedge p(2,j) \wedge \neg p(3,j)) \vee (p(1,j) \wedge \neg p(2,j) \wedge p(3,j)) \vee (\neg p(1,j) \wedge p(2,j) \wedge p(3,j))$$

2. Given that

$$r \wedge (\neg(\neg q \rightarrow p))$$

   is true, what are the truth values of :

(a) $q \rightarrow \neg r$

(b) $\neg r \wedge q$

(c) $p \iff \neg r$

(d) $\neg q \rightarrow \neg p$

OK, so, if this formula is true then we must have $r = T$ as well as $(\neg(\neg q \rightarrow p)) = T$. But then, it must be that $(\neg q \rightarrow p) = F$ We know that $\neg q \rightarrow p = q \vee p$, and for that to be F, both p and q must be F. So we know the truth values of all variables. Plugging them in, we have all but (b) evaluating to T.

3. Simplify $(p \rightarrow (\neg p \iff p))$ as much as you can.

   $(p \rightarrow (\neg p \iff p)) = \neg p \vee (\neg p \iff p)$.

   Looking at the second term,

   $\neg p \iff p = (\neg p \rightarrow p) \wedge (p \rightarrow \neg p) = (p \vee p) \wedge (\neg p \vee \neg p) = p \wedge \neg p = F$

   Then, $(p \rightarrow (\neg p \iff p)) = \neg p \vee F = \neg p$.

4. Simplify $(\neg(p \wedge q) \wedge (p \rightarrow q))$ as much as you can.

   Working on both terms simultaneously, we have

   $(\neg(p \wedge q) \wedge (p \rightarrow q)) = (\neg p \vee \neg q) \wedge (\neg p \vee q) = \neg p \vee (\neg q \wedge q) = \neg p \vee F = \neg p$

5. If $((\neg p \rightarrow q) \vee r)$ is false then what is the value of $\neg q \rightarrow p$?

   If this is the case, then we must have $r = F$ as well as $\neg p \rightarrow q = F$. But $\neg p \rightarrow q = p \vee q$, and for that to be F we need $p = q = F$. Plugging in, we have $\neg q \rightarrow p = F$.