Probabilistic programming using first-class stores and first-class continuations

Oleg Kiselyov
FNMOC
oleg@pobox.com

Chung-chieh Shan
Rutgers University
ccshan@rutgers.edu

ML workshop
September 26, 2010

Probabilistic inference

I have exactly two kids. At least one is a girl.

What is the probability that my older kid is a girl?

Probabilistic inference

Model (what) | Inference (how)

$$\left.\begin{array}{l}\Pr(\text{Reality}) \\ \text{Reality} \rightarrow \text{Obs}, \text{Result} \\ \text{obs}\end{array}\right\} \Pr(\text{Result} \mid \text{Obs} = \text{obs})$$

I have exactly two kids.
At least one is a girl.

What is the probability that
my older kid is a girl?

Declarative probabilistic inference

Model (what)	Inference (how)
$\mathrm{Pr}(\mathrm{Reality})$ $\mathrm{Reality} \to \mathrm{Obs}, \mathrm{Result}$ obs	$\left.\right\}$ $\mathrm{Pr}(\mathrm{Result} \mid \mathrm{Obs} = \mathrm{obs})$
I have exactly two kids. At least one is a girl.	What is the probability that my older kid is a girl?

Declarative probabilistic inference (UAI 2009, DSL 2009)

Model (what) Inference (how)

$$\left.\begin{array}{l}\Pr(\text{Reality})\\ \text{Reality} \to \text{Obs}, \text{Result}\\ \text{obs}\end{array}\right\} \Pr(\text{Result} \mid \text{Obs} = \text{obs})$$

I have exactly two kids. At least one is a girl.

What is the probability that my older kid is a girl?

Models and inference as interacting programs in the same general-purpose language

Declarative probabilistic inference (UAI 2009, DSL 2009)

Model (what) | Inference (how)

$$\left.\begin{array}{l}\mathrm{Pr}(\mathrm{Reality})\\ \mathrm{Reality} \to \mathrm{Obs}, \mathrm{Result}\\ \mathrm{obs}\end{array}\right\} \mathrm{Pr}(\mathrm{Result} \mid \mathrm{Obs} = \mathrm{obs})$$

I have exactly two kids.
At least one is a girl.

What is the probability that
my older kid is a girl?

```
let flip = fun p ->
  dist [(p, true);
        (1.-.p, false)]
in let girl1 = flip 0.5 in
   let girl2 = flip 0.5 in
   if girl1 || girl2
   then girl1 else fail ()
```

Models and inference
as interacting programs
in the same general-
purpose language

Declarative probabilistic inference (UAI 2009, DSL 2009)

Model (what) | Inference (how)

$$\left.\begin{array}{l}\Pr(\text{Reality})\\ \text{Reality} \to \text{Obs}, \text{Result}\\ \text{obs}\end{array}\right\} \Pr(\text{Result} \mid \text{Obs} = \text{obs})$$

I have exactly two kids.
At least one is a girl.

What is the probability that
my older kid is a girl?

```
normalize (exact_reify (fun () ->
let flip = fun p ->
  dist [(p, true);
        (1.-.p, false)]
in let girl1 = flip 0.5 in
   let girl2 = flip 0.5 in
   if girl1 || girl2
   then girl1 else fail ()))
```

true	1/2
false	1/4

Models and inference
as interacting programs
in the same general-
purpose language

Declarative probabilistic inference (UAI 2009, DSL 2009)

Model (what) — Inference (how)

$$\left.\begin{array}{l}\Pr(\text{Reality})\\ \text{Reality} \to \text{Obs}, \text{Result}\\ \text{obs}\end{array}\right\} \Pr(\text{Result} \mid \text{Obs} = \text{obs})$$

I have exactly two kids. At least one is a girl.

What is the probability that my older kid is a girl?

```
normalize (exact_reify (fun () ->
let flip = fun p ->
  dist [(p, true);
        (1.-.p, false)]
in let girl1 = flip 0.5 in
   let girl2 = flip 0.5 in
   if girl1 || girl2
   then girl1 else fail ()))
```

true	$2/3$
false	$1/3$

true	$1/2$
false	$1/4$

Models and inference as interacting programs in the same general-purpose language

Declarative probabilistic inference (UAI 2009, DSL 2009)

Model (what) — Inference (how)

$$\left.\begin{array}{l}\mathrm{Pr}(\mathrm{Reality})\\ \mathrm{Reality} \to \mathrm{Obs}, \mathrm{Result}\\ \mathrm{obs}\end{array}\right\} \mathrm{Pr}(\mathrm{Result} \mid \mathrm{Obs} = \mathrm{obs})$$

I have exactly two kids.
At least one is a girl.

What is the probability that
my older kid is a girl?

```
normalize (exact_reify (fun () ->
let flip = fun p ->
  dist [(p, true);
        (1.-.p, false)]
in let girl1 = flip 0.5 in
   let girl2 = flip 0.5 in
   if girl1 || girl2
   then girl1 else fail ()))
```

Expressive models and efficient inference as interacting programs in the same general-purpose language

Outline

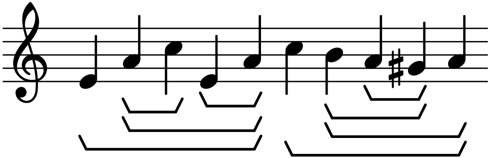
- **Expressive models**
    - Reuse existing infrastructure
    - Nested inference
- Efficient inference
    - First-class continuations
    - First-class stores

Motivic development in Beethoven sonatas (Pfeffer 2007)

Source motif

Motivic development in Beethoven sonatas (Pfeffer 2007)

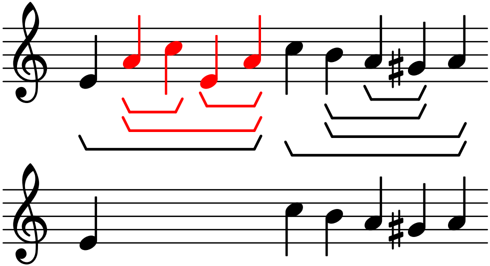
Source motif



Motivic development in Beethoven sonatas

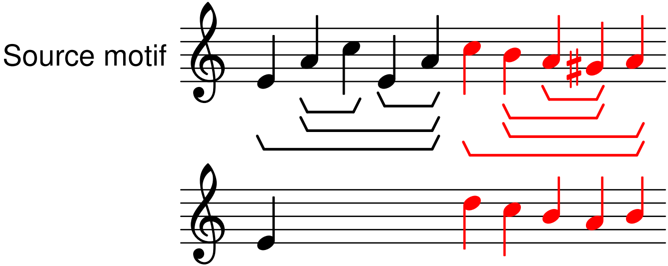
(Pfeffer 2007)

Source motif

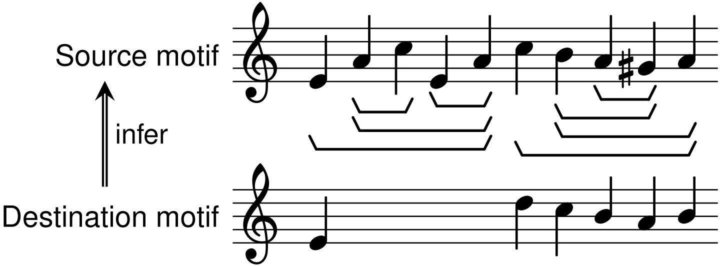


Motivic development in Beethoven sonatas

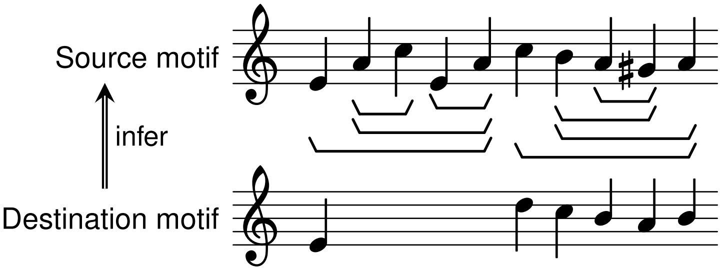
(Pfeffer 2007)



Motivic development in Beethoven sonatas (Pfeffer 2007)



Motivic development in Beethoven sonatas (Pfeffer 2007)

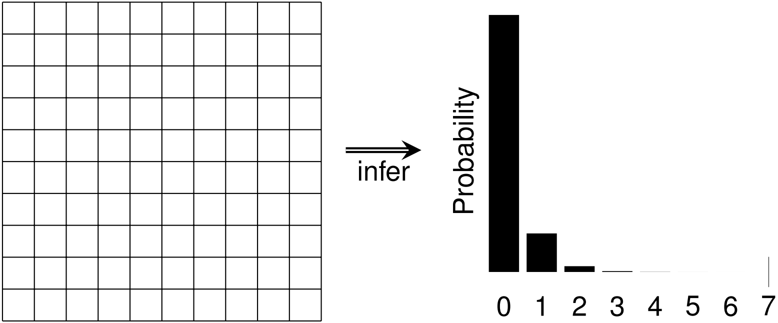


Motif pair		1	2	3	4	5	6	7
% correct								
Pfeffer 2007	(30 sec)	93	100	28	80	98	100	63
HANSEI	(90 sec)	98	100	29	87	94	100	77
HANSEI	(30 sec)	92	99	25	46	72	95	61

Importance sampling using lazy stochastic lists.

Noisy radar blips for aircraft tracking

(Milch et al. 2007)

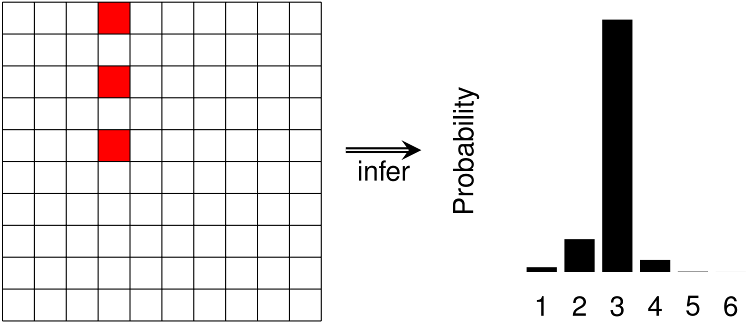


Blips present and absent

Number of planes

Noisy radar blips for aircraft tracking

(Milch et al. 2007)



Blips present and absent

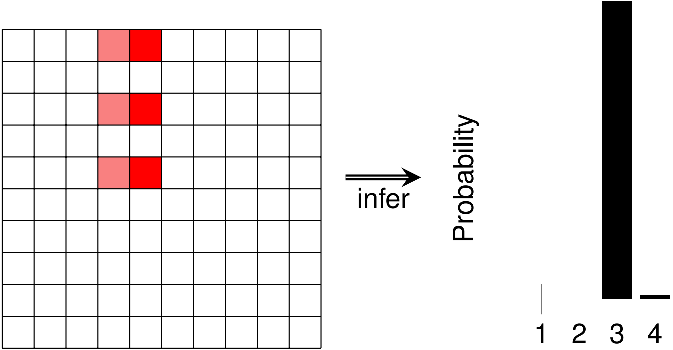
$t = 1$

Number of planes

Particle filter using lazy stochastic coordinates.

Noisy radar blips for aircraft tracking

(Milch et al. 2007)



Blips present and absent

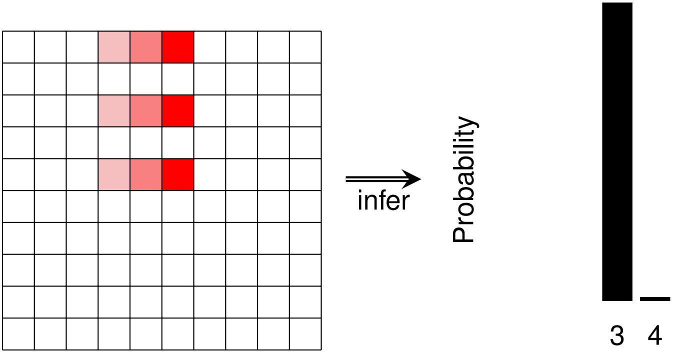
$t = 1, \quad t = 2$

Number of planes

Particle filter using lazy stochastic coordinates.

Noisy radar blips for aircraft tracking

(Milch et al. 2007)



Blips present and absent

$t = 1, \quad t = 2, \quad t = 3$

Number of planes

Particle filter using lazy stochastic coordinates.

Models as programs in a general-purpose language

Reuse existing infrastructure!

- Rich libraries: lists, arrays, database access, I/O, ...
- Type system
- Functions as first-class values
- Compiler
- Debugger
- Memoization

Implemented independently in Haskell, Scheme, Ruby, Scala ...

Models that invoke nested inference

Choose a coin that is either fair or completely biased for `true`.

```
let biased = flip 0.5 in
let coin = fun () -> flip 0.5 || biased in
```

Models that invoke nested inference

Choose a coin that is either fair or completely biased for `true`.

```
let biased = flip 0.5 in
let coin = fun () -> flip 0.5 || biased in
```

Let p be the probability that flipping the coin yields `true`.

What is the probability that p is at least 0.3?

Models that invoke nested inference

Choose a coin that is either fair or completely biased for `true`.

```
let biased = flip 0.5 in
let coin = fun () -> flip 0.5 || biased in
```

Let p be the probability that flipping the coin yields `true`.

What is the probability that p is at least 0.3?
Answer: 1.

```
at_least 0.3 true (exact_reify coin)
```

Models that invoke nested inference

```
exact_reify (fun () ->
```

Choose a coin that is either fair or completely biased for `true`.

```
let biased = flip 0.5 in
let coin = fun () -> flip 0.5 || biased in
```

Let p be the probability that flipping the coin yields `true`.

What is the probability that p is at least 0.3?
Answer: 1.

```
at_least 0.3 true (exact_reify coin)          )
```

Models that invoke nested inference

```
exact_reify (fun () ->
```

Choose a coin that is either fair or completely biased for `true`.

```
let biased = flip 0.5 in
let coin = fun () -> flip 0.5 || biased in
```

Let p be the probability that flipping the coin yields `true`.
Estimate p by flipping the coin twice.
What is the probability that our estimate of p is at least 0.3?
Answer: 7/8.

```
at_least 0.3 true (sample 2 coin)            )
```

Models that invoke nested inference

```
exact_reify (fun () ->
```

Choose a coin that is either fair or completely biased for `true`.

```
let biased = flip 0.5 in
let coin = fun () -> flip 0.5 || biased in
```

Let p be the probability that flipping the coin yields `true`.
Estimate p by flipping the coin twice.
What is the probability that our estimate of p is at least 0.3?
Answer: 7/8.

```
at_least 0.3 true (sample 2 coin)            )
```

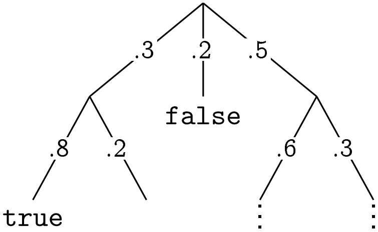
Returns a distribution, using `dist` like models do.
Works with observation, recursion, memoization.
Metareasoning without interpretive overhead.

Outline

Expressive models
- Reuse existing infrastructure
- Nested inference

▶ **Efficient inference**
- First-class continuations
- First-class stores

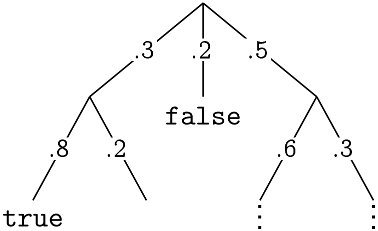
Reifying a model into a lazy search tree



not syntax tree
not call tree

```
type 'a branch = V of 'a | C of (unit -> 'a tree)
and  'a tree   = (prob * 'a branch) list
```

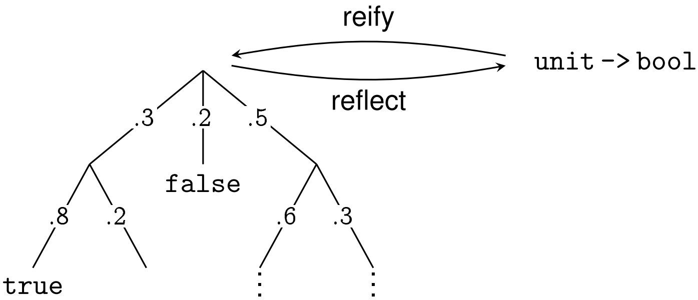
Reifying a model into a lazy search tree



not syntax tree
not call tree

Depth-first enumeration = exact inference
Random dive = rejection sampling
Dive with look-ahead = importance sampling

Reifying a model into a lazy search tree

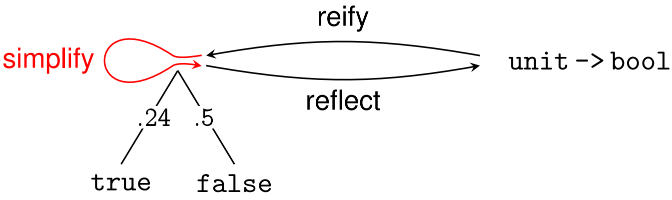


Represent a probability and state monad (Filinski 1994)
using first-class delimited continuations, aka clonable threads:

- Model runs inside a thread.
- `dist` clones the thread.
- `fail` kills the thread.

Models' code stays opaque. Deterministic parts run at full speed.
Nesting works.

Reifying a model into a lazy search tree



reflect ∘ simplify ∘ reify = table, chart, bucket
reflect ∘ sample ∘ reify = particle filter

The library so far

```
type 'a branch = V of 'a | C of (unit -> 'a tree)
and  'a tree   = (prob * 'a branch) list
```

```
let reify m = reset (fun () -> [(1.0, V (m ()))])

let dist ch = shift (fun k ->
  List.map (fun (p,v) -> (p, C (fun () -> k v))) ch)
```

The library so far

```
type 'a branch = V of 'a | C of (unit -> 'a tree)
and  'a tree   = (prob * 'a branch) list

let prompt = new_prompt ()

let reify m = reset prompt (fun () -> [(1.0, V (m ()))])

let dist ch = shift prompt (fun k ->
  List.map (fun (p,v) -> (p, C (fun () -> k v))) ch)
```

First-class continuations

```
type req = Done | Choice of (prob * (unit -> req)) list

let reify m =
  let answer = ref None in
  let rec interp req = match req with
    | Done ->
        let Some v = !answer in [(1.0, V v)]
    | Choice ch ->
        List.map (fun (p,m) ->
                        (p, C (fun () -> interp (m ()))))
                  ch
  in interp (reset prompt (fun () ->
                answer := Some (m ()); Done))

let dist ch = shift prompt (fun k ->
  Choice (List.map (fun (p,v) -> (p, fun () -> k v)) ch))
```

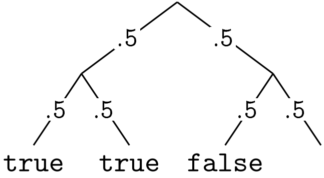
Memoization

```
type gender = Female | Male

let kid = memo (fun n -> dist [(0.5, Female);
                               (0.5, Male)])
in if kid 1 = Female || kid 2 = Female
   then kid 1 else fail ())
```

Memoization

```
type gender = Female | Male

let kid = memo (fun n -> dist [(0.5, Female);
                                (0.5, Male)])
in if kid 1 = Female || kid 2 = Female
   then kid 1 else fail ())
```

Used to speed up inference (ICFP 2009)

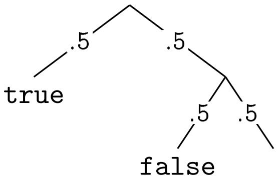


by delaying choices until observed

Memoization

```
type gender = Female | Male

let kid = memo (fun n -> dist [(0.5, Female);
                               (0.5, Male)])
in if kid 1 = Female || kid 2 = Female
   then kid 1 else fail ())
```

Used to speed up inference (ICFP 2009)



by delaying choices until observed

Memoization

```
type gender = Female | Male

let kid = memo (fun n -> dist [(0.5, Female);
                               (0.5, Male)])
in if kid 1 = Female || kid 2 = Female
   then kid 1 else fail ())
```

Used to speed up inference (ICFP 2009)
and to express nonparametric distributions (Goodman et al. 2008)

Lazy evaluation is `memo (fun () -> ...)`

Each search-tree node must keep its own store ('thread-local')
Nesting creates *regions* of memo cells (ICFP 2006)

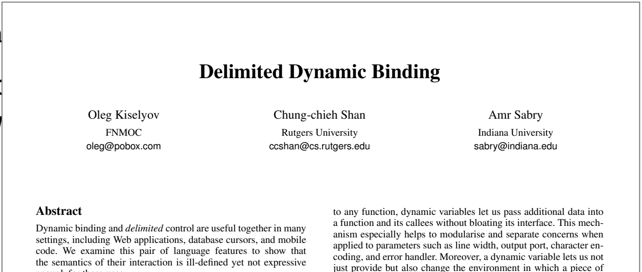
Memoization

```
type gender = Female | Male

let kid = memo (fun n -> dist [(0.5, Female);
                               (0.5, Male)])
in if kid 1 = Female || kid 2 = Female
   then kid 1 else fail ())
```

Used to speed up inference (ICFP 2009)
and to express nonparametric distributions (Goodman et al. 2008)

Lazy evaluation is m

Each search-tree no
Nesting creates *reg*



Delimited Dynamic Binding

Oleg Kiselyov
FNMOC
oleg@pobox.com

Chung-chieh Shan
Rutgers University
ccshan@cs.rutgers.edu

Amr Sabry
Indiana University
sabry@indiana.edu

Abstract

Dynamic binding and *delimited* control are useful together in many settings, including Web applications, database cursors, and mobile code. We examine this pair of language features to show that the semantics of their interaction is ill-defined yet not expressive

to any function, dynamic variables let us pass additional data into a function and its callees without bloating its interface. This mechanism especially helps to modularise and separate concerns when applied to parameters such as line width, output port, character encoding, and error handler. Moreover, a dynamic variable lets us not just provide but also change the environment in which a piece of

First-class stores: interface

```
module Memory = struct
  type 'a loc
  type t
  val newm : t
  val new_loc : unit -> 'a loc
  val mref : 'a loc -> t -> 'a  (* throws Not_found *)
  val mset : 'a loc -> 'a -> t -> t
end
```

First-class stores: usage

```
let reify m =
  let answer = ref None in
  let rec interp req = match req with
    | Done ->
        let Some v = !answer in [(1.0, V v)]
    | Choice ch ->
        List.map (fun (p,m) ->
                    (p, C (fun () -> interp (m ()))))
          ch
  in
  let mem = !thread_local in
  thread_local := Memory.newm;
  let req = reset prompt (fun () ->
      answer := Some (m ()); Done) in
  thread_local := mem;
  interp req
```

Recap

Expressive models and efficient inference
as interacting programs
in the same general-purpose language

We want first-class delimited continuations and
(garbage-collector support for) first-class stores

HANSEI `http://okmij.org/ftp/kakuritu/`

Recap

Expressive models and efficient inference
as interacting programs
in the same general-purpose language

We want first-class delimited continuations and
(garbage-collector support for) first-class stores

HANSEI `http://okmij.org/ftp/kakuritu/`