Monolingual probabilistic programming using generalized coroutines

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This session . . .

programming

formalism

This talk . . .

Modular programming

Expressive formalism

Efficient implementation

This talk … is about knowledge representation

Modular programming	–	Factored representation
Expressive formalism	–	Informative prior
Efficient implementation	–	Custom inference

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Declarative probabilistic inference

Model (what)	Inference (how)

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Toolkit (BNT, PFP)	invoke →	distributions, conditionalization, …
Language (BLOG, IBAL, Church)	random choice, observation, …	← interpret

Declarative probabilistic inference

	Model (what)	Inference (how)
Toolkit (BNT, PFP)	+ use existing libraries, types, debugger	+ easy to add custom inference
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Express models and inference as interacting programs in the same general-purpose language.

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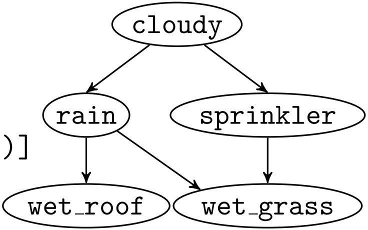
Today: Best of both	Payoff: expressive model + models *of inference*: bounded-rational theory of mind	Payoff: fast inference + deterministic parts of models run *at full speed* + importance sampling

Express models and inference as interacting programs in the same general-purpose language.

Outline

- **Expressivity**
  - Memoization
  - Nested inference
- Implementation
  - Reifying a model into a search tree
  - Importance sampling with look-ahead
- Performance

Grass model

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let flip = fun p ->
 dist [(p, true); (1.-.p, false)]
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Models are ordinary code (in OCaml) using a library function `dist`.

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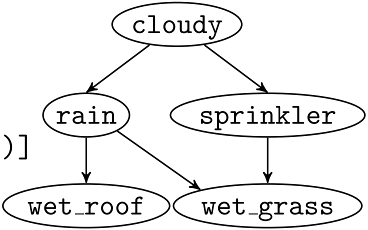


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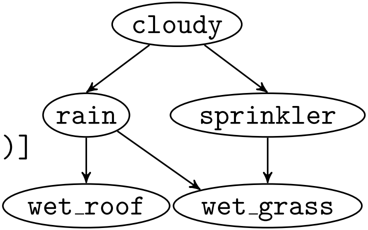


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 let cloudy    = flip 0.5 in
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 let wet_roof  = flip 0.7 && rain in
 let wet_grass = flip 0.9 && rain ||
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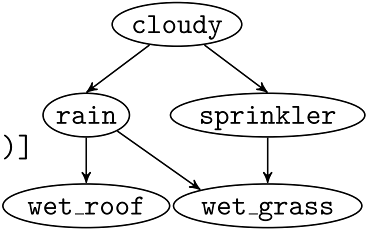


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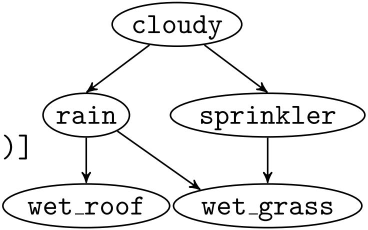


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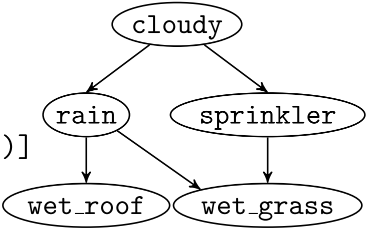


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Models are ordinary code (in OCaml) using a library function `dist`.
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Inference applies to *thunks* and returns a distribution.
Deterministic parts of models run at full speed.

Models as programs in a general-purpose language

Reuse existing infrastructure!

- Rich libraries: lists, arrays, database access, I/O, . . .
- Type inference
- Functions as first-class values
- Compiler
- Debugger
- Memoization

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  Express Dirichlet processes, etc. (Goodman et al. 2008)

  Speed up inference using lazy evaluation

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  Speed up inference using lazy evaluation
bucket elimination
sampling w/memoization (Pfeffer 2007)

Nested *inference*

Choose a coin that is either fair or completely biased for `true`.

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Answer: 1.

```
at_least 0.3 true (exact_reify coin)
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Nested *inference*

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exact_reify (fun () ->
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Estimate p by flipping the coin twice.
What is the probability that our estimate of p is at least 0.3?
Answer: 7/8.

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Returns a distribution—not just nested `query` (Goodman et al. 2008).
Inference procedures are OCaml code using `dist`, like models.
Works with observation, recursion, memoization.
Bounded-rational theory of mind without interpretive overhead.

Outline

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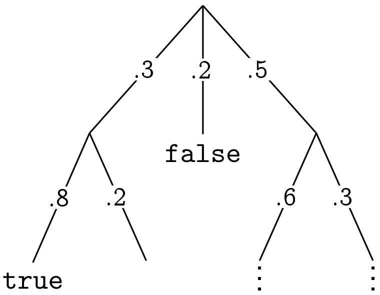
  - Memoization
  - Nested inference
- ▶ **Implementation**
  - Reifying a model into a search tree
  - Importance sampling with look-ahead
- Performance

Reifying a model into a search tree



Exact inference by depth-first brute-force enumeration.
Rejection sampling by top-down random traversal.

Reifying a model into a search tree

open

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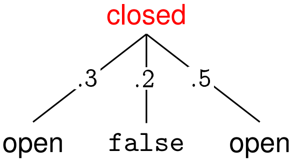

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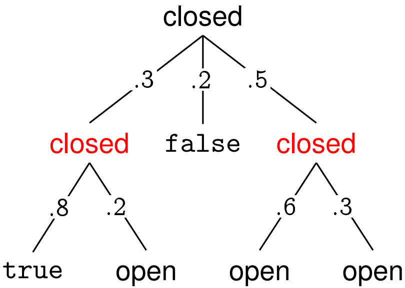

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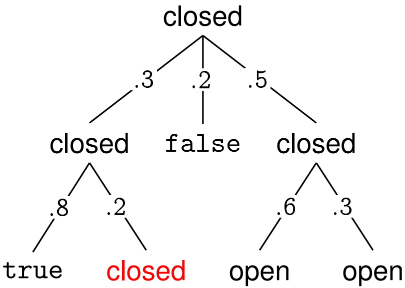

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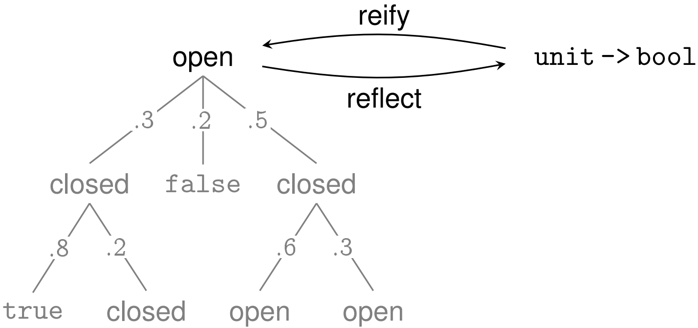


Inference procedures cannot access models' source code.

Reify then reflect:

- Brute-force enumeration becomes bucket elimination
- Sampling becomes particle filtering

Reifying a model into a search tree

$$\begin{tikzcd} \text{open} \arrow[r, bend right, "\text{reflect}"'] & \texttt{unit -> bool} \arrow[l, bend right, "\text{reify}"'] \end{tikzcd}$$

Implementation: represent a probability and state monad
(Giry 1982, Moggi 1990, Filinski 1994)
using first-class delimited continuations
(Strachey & Wadsworth 1974,
Felleisen et al. 1987,
Danvy & Filinski 1989)

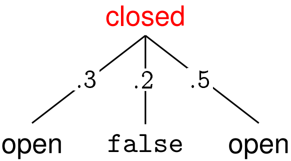
Implementation: using clonable user-level threads

- Model runs inside a thread.
- `dist` clones the thread.
- `fail` kills the thread.
- Memoization mutates thread-local storage.

Importance sampling with look-ahead

open Probability mass $p_c = 1$

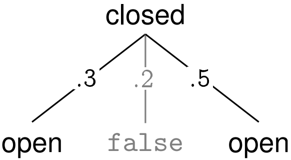
Importance sampling with look-ahead



Probability mass $p_c = 1$

1. Expand one level.

Importance sampling with look-ahead

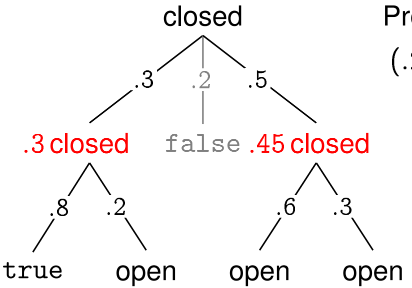


Probability mass $p_c = 1$

$(.2, \texttt{false})$

1. Expand one level.
2. Report shallow successes.

Importance sampling with look-ahead



Probability mass $p_c = .75$

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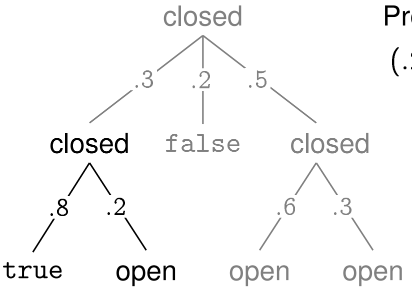
2. Report shallow successes.
3. Expand one more level and tally open probability.

Importance sampling with look-ahead



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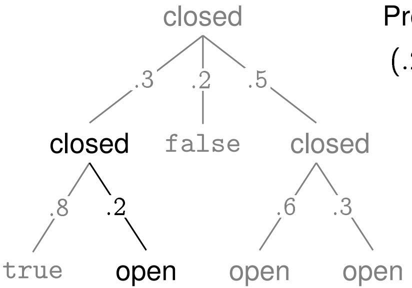

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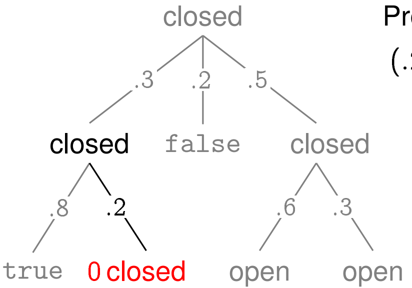

Probability mass $p_c = 0$

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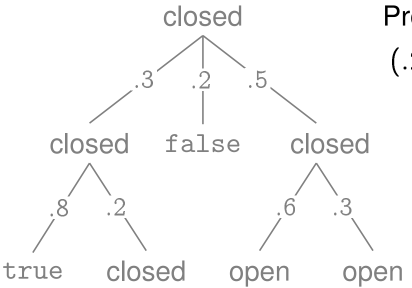

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▶ **Performance**

Motivic development in Beethoven sonatas

(Pfeffer 2007)

Source motif

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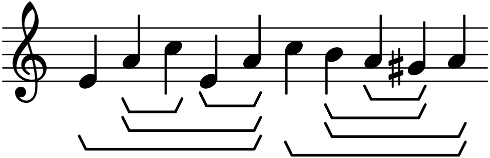

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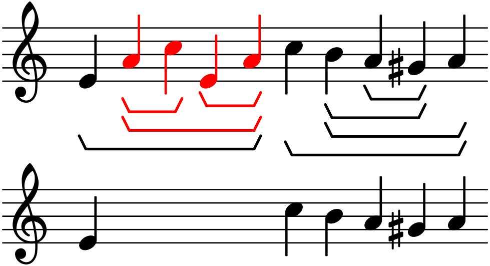

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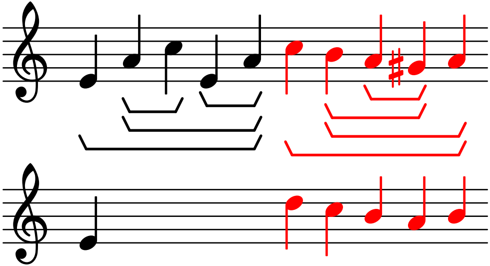

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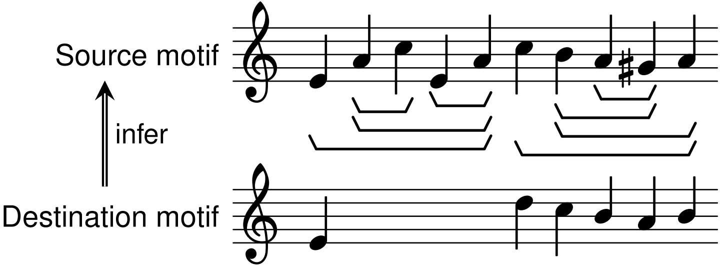

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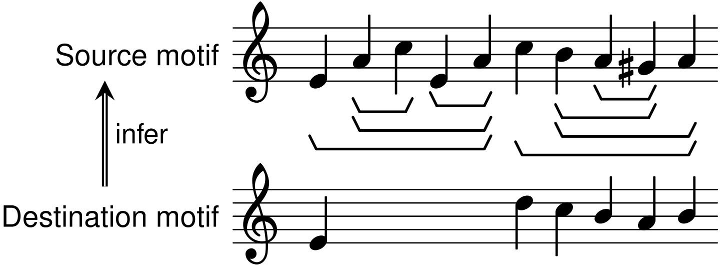

Implemented using lazy stochastic lists.

Motif pair	1	2	3	4	5	6	7
% correct using importance sampling							
• Pfeffer 2007 (30 sec)	93	100	28	80	98	100	63
• This paper (90 sec)	98	100	29	87	94	100	77
• This paper (30 sec)	92	99	25	46	72	95	61

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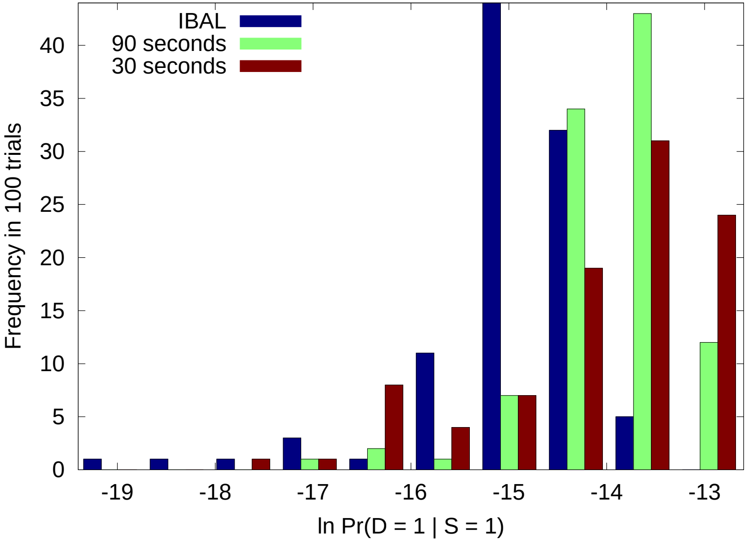
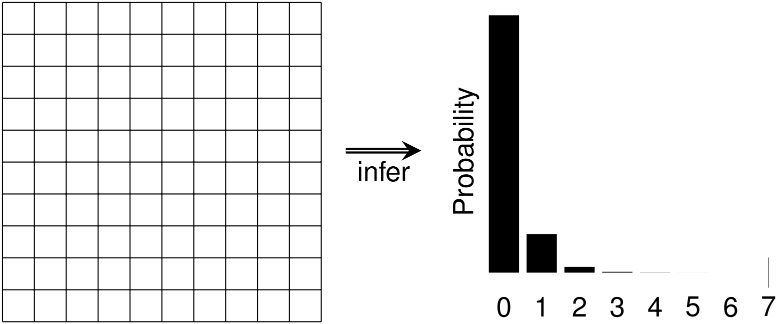


Noisy radar blips for aircraft tracking

(Milch et al. 2007)



Blips present and absent

Number of planes

Particle filter. Implemented using lazy stochastic coordinates.

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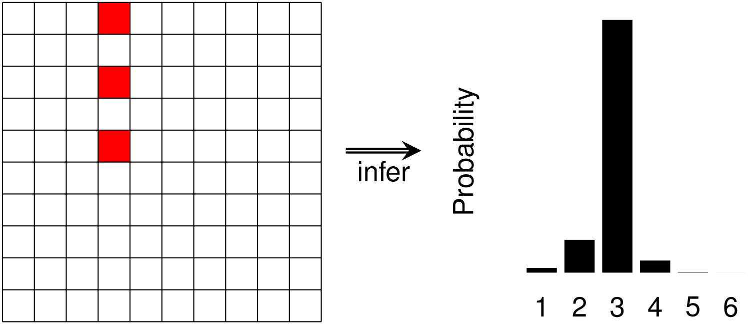

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$t = 1$

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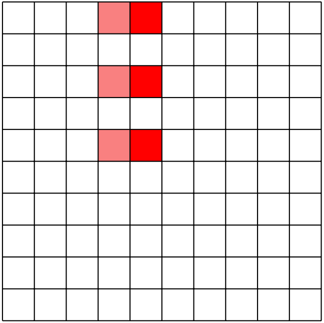
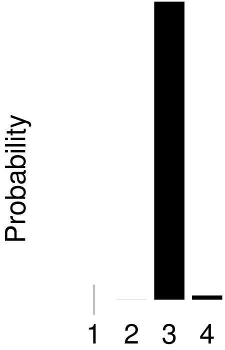


Blips present and absent

$t = 1, \quad t = 2$

$\Longrightarrow$ infer



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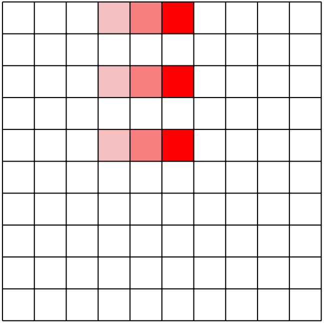

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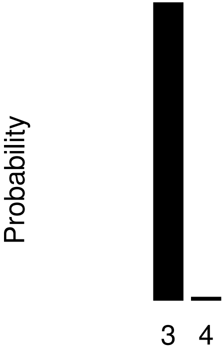

Blips present and absent

$t = 1, \quad t = 2, \quad t = 3$

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Language	+ random variables are ordinary variables	+ compile models for faster inference
Today: Best of both	Payoff: expressive model + models *of inference*: bounded-rational theory of mind	Payoff: fast inference + deterministic parts of models run *at full speed* + importance sampling

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